First Exam MTH 221 , summer 2011

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QUESTION 1. Each = 2.5 points, Circle the correct answer

(i) If A is a 3×3 matrix and det(A) = 2, then det(3A) =

a. 6

b. $\frac{3}{2}$

c. 24

d. 54

e. None of the above

(ii) If A is a 3×3 matrix such that $det(A) = -0.5$, then the system $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

a. has infinitely many solution and $x_1 = x_2 = x_3 = 0$ is a solution to the system.

b. is possible to be inconsistent

c. $x_1 = x_2 = x_3 = -0.5$ is a solution to the system

d. $x_1 = x_2 = x_3 = 0$ is the only solution to the system

(iii) If A, B are 4×4 matrices such that $det(2A) = det(B) = 8$. Then $det(ABA^T) =$

a. 512

b. 128

c. 32

d. 2

e. none of the above

(iv) Let A be a 5×5 matrix such that the third column and the fifth column of A are identical. Let B be the third column of A. Then one of the following statement is correct:

a. $x_1 = x_2 = x_4 = 0$, $x_3 = x_5 = 1$ is a solution to the system $AX = B$.

b. $x_1 = x_2 = x_4 = 0$,$x_3 = x_5 = 0.5$ is the only solution to the system $AX = B$.

c. It is possible that the system $AX = B$ is inconsistent

d. $x_1 = x_2 = x_4 = 0$, $x_3 = \pi$, $x_5 = 1 - \pi$ is a solution to the system.

(v) Let A be a 2×2 matrix and $B = \begin{bmatrix} a_1 & a_2 \\ a_1 & 0 \end{bmatrix}$.Given $AB = \begin{bmatrix} 3a_1 & 2a_2 \\ 0 & -a_2 \end{bmatrix}$. Then $A =$

a. $\begin{bmatrix} 3 & 0 \\ -1 & 0 \end{bmatrix}$.

b. $\begin{bmatrix} 3 & 0 \\ -1 & 1 \end{bmatrix}$.

c. $\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$.

d. $\begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix}$.

(vi) let $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$. Then A^{-1}

a. $\begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 0 & -1 & -1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d. None of the above

(vii) Let A as above. Then a solution to $A^T X = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$ is

a. $x_1 = x_2 = 0, x_3 = 1$

b. $x_1 = 1, x_2 = -1, x_3 = 0$

c. the system has infinitely many solution and $x_1 = -1, x_2 = -1, x_3 = 0$ is a solution

d. none of the above

(viii) The values of k which make the system $\begin{bmatrix} 1 & 1 & 1 \\ -1 & -1 & (k-1) \\ -2 & -2 & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k \\ 1-k \\ -k \end{bmatrix}$ consistent are

a. k = -1 and k = 2

b. $k \neq 0$ and $k \neq -2$

c. $k \neq 1$ and $k \neq 2$

d. none of the above.

(ix) The values of k which make the system $\begin{bmatrix} 2 & 0 & 1 \\ -2 & 1 & 1 \\ -2 & -2 & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ 4 \end{bmatrix}$ inconsistent are

a. $k = -5$

b. $k = -1$

c. $k = -3$

d. None of the above

(x) The values of k and a that make the system $\begin{bmatrix} 2 & 2 & 8 \\ -2 & -1 & 1 \\ -2 & -2 & k \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} a \\ 3 \\ 4 \end{bmatrix}$ inconsistent are

a. $k = -8$ and $a \neq -4$

b. $k = -8$ and $a = -4$

c. $k = -4$ and $a \in R$

d. none of the above

(xi) Let $A = \begin{bmatrix} -5 & -2 \\ 6 & 2 \end{bmatrix}$. Then $A^{-1} =$

a. $\begin{bmatrix} 1 & 1 \\ -3 & -2.5 \end{bmatrix}$. b. $\frac{-1}{22} \begin{bmatrix} 2 & 2 \\ -6 & -5 \end{bmatrix}$. c. $\frac{-1}{22} \begin{bmatrix} 2 & -6 \\ 2 & -5 \end{bmatrix}$. d. none of the above

(xii) Let A be a 3×3 matrix and C be the completely reduced form of A such that

$$A \ \overrightarrow{R_1 \leftrightarrow R_2} \ A_1 \ \overrightarrow{2R_1 + R_3 \to R_3} \ A_2 \ \overrightarrow{2R_3} \ C = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ Then the solution to the system $AX = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

a. $x_1 = 1, x_2 = 2, x_3 = 4$ b. $x_1 = 2, x_2 = 1, x_3 = 4$ c. $x_1 = 1, x_2 = 0.5, x_3 = 0$
d. None of the above

(xiii) Consider question (xii). A^{-1} is

a. $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 4 & 2 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 2 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ d. None of the previous is correct

(xiv) Consider question (xii). $det(A)$ is

a. 2 b. 0.5 c. -2 d. None of the previous is correct.

(xv) Consider question (xii). Let E be an elementary matrix such that $EA_2 = A_1$. Then E is

a. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(xvi) Let $A = \begin{bmatrix} 2 & 1 & -1 & 2 \\ -2 & 1 & 4 & 6 \\ -4 & -2 & 3 & 4 \\ -2 & -1 & 2 & 1 \end{bmatrix}$. Then $det(A) =$

a. 20 b. 12 c. -20 d. None of the previous is correct

QUESTION 2. (Each = 2 points) Answer the following as true or false: NO WORKING NEED BE SHOWN.

(i) If A is a 3×3 matrix and the system $AX = \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix}$ has infinitely many solutions, then $det(A) = 0$.

(ii) If A is a 5×5 matrix and the system $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ has only the trivial solution, then $det(A) \neq 0$.

(iii) If A is a 4×5 matrix and the system $AX = \begin{bmatrix} 1 \\ 0 \\ 3 \\ 5 \end{bmatrix}$ is consistent, then it has infinitely many solutions.

(iv) If A is a 3×3 matrix and A is row-equivalent to B, then there exists an invertible matrix F such that $FA = B$.

(v) If A, B are 6×6 matrices and $AB \neq BA$, then it is possible that $det(AB) \neq det(BA)$.

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